On Designing Optimal Data Purchasing Strategies for Online Ad Auctions*

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ABSTRACT

In online advertising, advertisers can purchase consumer relevant data from data marketplaces with a certain expenditure, and exploit the purchased data to guide the bidding process in ad auctions. One of the pressing problem faced by advertisers is to design the optimal data purchasing strategy (how much data to purchase to be competitive in bidding process) in online ad auctions. In this paper, we model the data purchasing strategy design as a convex optimization problem, jointly considering the expenditure paid during data purchasing and the benefits obtained from ad auctions. Using the techniques from Baysian game theory and convex analysis, we derive the optimal purchasing strategies for advertisers in different market scenarios. We also theoretically prove that the resulting strategy profile is the unique one that achieves Nash Equilibrium. Our analysis shows that the proposed data purchasing strategy can handle diverse ad auctions and valuation learning models. Our numerical results empirically reveal how the equilibrium state changes with variation of the strategic environment.

CCS CONCEPTS

• **Theory of computation → Computational advertising theory**; *Algorithmic game theory*; *Solution concepts in game theory*;

KEYWORDS

Ad Auctions; Targeting; Information Acquisition

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1 INTRODUCTION

Targeting is a technique to enable advertisers to deploy advertising campaigns on the consumers from certain market segments, such that the advertisers can spend their finite ad budgets on the most relevant consumer. It is difficult to conduct and evaluate a qualified advertising without enough consumer relevant data. Fortunately, with the advance of online tracking techniques, the advertisers now can collect a large amount of relevant data, such as third-party-cookies [6, 19, 26, 49], to build the profiles of consumers, and then conduct accurate targeted advertising.

The consumer relevant data is currently traded over the Internet. The collection and distribution of consumer relevant data are conducted by Data Management Platforms, ranging from well-known data analysis companies such as Acxiom[1] and Bloomberg[2], to emerging companies such as Bluekai[3] and eXelate[4]. Data Management Platforms create online marketplaces, where these companies can upload consumers relevant data to make profits, and advertisers can purchase the desired datasets to enable targeted advertising. The marketing demand for such highly detailed, consumer-level data is mostly driven by advertising industries.

In advertising ecosystem, major search engines are now leveraging auctions as main monetization channels [16, 21, 32, 42], including forms of sponsored search auctions [31, 39, 42, 43] and realtime bidding (RTB) [12, 14, 50]. In sponsored search, a selective set of ads related to the user query will be shown together with returned relevant webpages after in the search engine, while In RTB for display advertising, an ad impression with related information will be sent to advertisers through the ad exchange when the user visits the website. For both scenarios, auctions are held and bids are collected to determine the ad allocations and corresponding charges. However, the uncertainty of valuations over the ad slots, which may varies across advertisers, causes difficulties for launching successful ad campaigns. Without full information about the consumers, it is hard for advertisers to extract the precise valuations for the ad slots. Either underestimation or overestimation of valuations could lead to improper bidding strategies in ad auctions.

Therefore, Data Management Platforms have become demanding places for advertisers to refine their valuations by purchasing consumer relevant data. By buying enough amount of data, the advertisers can extract valuable information about the demographic and psychographic characteristics of consumers via the data mining techniques [19, 49], and further tailor their ad campaigns to their preferred consumers. While advertisers can learn more precise valuations from buying a larger amount of data, they also have to pay more money for such purchasing, or exert more efforts or energy to extract such valuable information. Hence, one of the pressing problems faced by advertisers is to design an optimal data purchasing strategy by making a trade-off between the expenditure paid during data purchasing and the expected utility increase in the auction.

There are several challenges in designing such a data purchasing strategy for online ad auctions. The first challenge comes from the various formats of ad auctions. The ultimate goal of an advertiser

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[1] Acxiom: http://www.acxiom.com/
[2] Bloomberg: http://www.bloomberg.com/
[3] Bluekai: http://www.bluekai.com/
[4] eXelate: http://exelate.com/

is to purchase an appropriate amount of data to maximize her expected utility in ad auctions. Thus, the data purchasing strategy design is highly related to the specific procedure of the auction. However, the variety of ad auctions in practice, such as Generalized Second Price (GSP), Generalized First Price (GFP) [21], and Vickrey-Clarke-Groves (VCG) mechanism [17, 29, 47], increases the difficulty in analyzing data purchasing strategies.

The second challenge comes from the diverse valuation learning models of advertisers. The data purchasing strategy design is to solve the payoff maximization problem under the strategic environment. The payoff of an advertiser is defined as the difference between the utility obtained from ad auction and the expenditure paid to purchase consumer relevant data. In order to extract true valuations and then obtain high utilities in ad auctions, advertisers may adopt diverse valuation learning models [33, 34, 44] upon the purchased data. Without specifying the learning procedure of other advertisers, an advertiser may not be possible to infer her competitors' data purchasing strategies, which significantly increases the difficulty of designing an optimal data purchasing strategy.

In this paper, we develop a framework to solve the optimal data purchasing strategy design problem, by jointly considering the above challenges. We first model the various ad auctions as Bayesian games with the same ad allocation rule. Using Payoff Equivalence Principle [38], we demonstrate that the expected utilities of advertisers are independent on the specific formats of ad auctions, decoupling the data purchasing stage from the auction stage. We then propose a data purchasing model to capture the diverse valuation learning models of advertisers, and formulate the optimal data purchasing strategy design as a convex optimization problem. Using the techniques from game theory and convex analysis, we can explicitly derive the optimal data purchasing strategy for advertisers, and theoretically prove that such a strategy profile is a unique Nash Equilibrium. Our numerical results further illustrate how would advertisers behave under various strategic environments. We summarize our key contributions in this work as follows.

• First, we propose a general framework consisting of an ad auction model and a data purchasing model. The framework is powerful enough to comprehend a variety of ad auction formats and different classes of learning agents, as well as to express the trade-offs advertisers have to consider when purchasing data. To the best of our knowledge, we are the first to study the data purchasing strategy design in an online ad auction setting.

• Second, we begin with considering a simple but representative case, where two Gaussian Learning agents compete for two different ad slots. We rigorously prove the existence and uniqueness properties of the Nash Equilibrium, as well as verify several intuitions of the equilibrium structure under both homogeneous and heterogeneous settings. Through this basic case, we demonstrate the rationale of finding the optimal data purchasing strategy.

• Third, we further extend this work by considering a more general scheme, where there can be a finite arbitrarily number of advertisers and slots. We show a general method to calculate the optimal strategy, and prove that the uniqueness and existence of the equilibrium are guaranteed given that the agents' learning processes satisfy a particular structure.

• Last but not least, we conduct a numerical study on two particular types of learning agents under our framework. We empirically reveal how much information will advertisers purchase under different strategic environments.

The rest of this paper is organized as follows. Section 2 provides the notations and the basic framework used throughout this paper.

Figure 1: The timing of the game.

In Section 3, we solve the optimal strategy design under a simple setting. In Section 4, we extend the model to a more general scheme and provide the corresponding theorems. Numerical results are provided in Section 5 to show how the different strategic environments affect the optimal strategies. Related works are reviewed in Section 6. We summarize our work in Section 7.

2 PRELIMINARIES AND PROBLEM FORMULATION

In this section, we develop models and notations used throughout this paper. Since we focus on the data purchasing strategy design in context of online advertising, which is related to the formats of ad auctions, we first present the ad auction model and then the data purchasing model.

As shown in Figure (1), we consider one round of ad auction which can be regarded as a two-stage game: it consists of a data purchasing (DP) stage and an ad auction stage. We will later specify that the first stage is a complete information game while the second is a Bayesian game. From now on we refer the advertisers as the agents in the model. First, the auctioneer announces the rules of ad auction. Next, agents purchase data from Data Market according to some strategies. After that, agents extract messages from purchased data, and refine their knowledge about their valuations over ad slots according to some learning model. Finally, all agents participate in the ad auction with their updated knowledge and receive the outcomes.

2.1 Ad Auction Model

There are N agents competing for $K \leq N$ ad slots. Denote ω_i as the valuation of agent i's ad for a click. In practice, there may be different classes of agents each round [11], and the valuation of one slot to different agents with various experience and identities is not fixed [2]. We capture these uncertainties by modeling that in prior, valuations of agents within the same class are identically distributed, while valuations of agents of different classes are independent [35, 41, 46]. We let $\eta(i)$ be the class of i, which is interpreted as the finest prior information to distinguish between the agents. In our framework $\eta(i)$ indicates: (1) the prior valuation distribution, and (2) the cost function (described in Section 2.2), of agent i. The prior distribution for agents of class $\eta(i)$ is denoted as $F_{\eta(i)}$, *i.e.*, $\omega_i \sim F_{\eta(i)}$. We assume the class information to be public prior knowledge, which is widely adopted by works regarding classical Bayesian game theory [28, 30]. Regarding her own valuation, we suppose i just knows as much as anybody else before purchasing data, *i.e.*, $F_{\eta_{(i)}}$. But after i having purchased and learned from data (targeting), from her point of view the knowledge of ω_i is updated from $F_{\eta_{(i)}}$ to a new distribution, which is not observed by others.

Every agent reports her bid b_i and therefore gets ranked by it. Then some agents win and obtain their positions from top to bottom according to their ranking, leaving those who lost unassigned. Each ad slot j has a corresponding click-through-rate (CTR) c_j. We restrict $c_j \equiv 0$ for $j > K$ and denote $\mathbf{c} = (c_1, c_2 \ldots c_N)^T$ as the CTR profile. In this paper, we assume $c_1 > c_2 > \ldots > c_K > 0$.

The auctioneer sorts the agents in descending order of their bids. The allocation rule can be represented as $x : \mathbb{R}^N \mapsto c^N$. More specifically, given bid profile $\mathbf{b} = (b_1, b_2 \ldots b_N)$, $x_i(\mathbf{b}) = c_j$ if and only if b_i is the j-th highest bid in $\mathbf{b}$ (ties are broken randomly). Then agent i's utility would be $u_i = \omega_i x_i(\mathbf{b}) - p_i(\mathbf{b})$, where $p_i(\mathbf{b})$ is her charge according to some payment rule.

The study of the equilibrium in ad auctions is of the central role in most works in this field. We formally define the bayesian view of equilibrium concept in our ad auction model as follows.

Definition 2.1 (Bayesian-Nash Equilibrium in Position Auction (BNE_{PA})). A profile of $(b_1^*, b_2^* \ldots b_N^*)$ forms a Bayesian-Nash Equilibrium in a position auction if $\forall i, b', \mathbb{E}[u_i(b_i^*, b_{-i}^*)] \geq \mathbb{E}[u_i(b', b_{-i}^*)]$.

The guarantee of equilibriums is closely related to the allocation rule, payment rule, and the distribution of agents [28]. However, analyzing the existence of equilibrium of a particular form of position auction is not our main focus in this work. We will assume that the mechanism announced by the auctioneer will always guarantee agents to reach a BNE_{PA}, which is formally defined as follows.

Definition 2.2 (Standard Position Auction (SPA)). A position auction is called an Standard Position Auction, if there always exists a BNE_{PA} regardless of the strategies in DP stage.

Examples of SPA include *laddered auction* proposed by [3] for its truthful dominant strategy. Generally speaking any position auctions with VCG-like payment rule are SPA for the same reason. However, things become complicated when coming to payment rules of GFP and GSP. The authors of [13] proved there exists only one symmetric BNE in a class of ad auctions representing by GFP. And [28] provided with a necessary and sufficient condition for GSP to have BNE in a symmetric setting. Leveraging Payoff Equivalence Principle [38], the expected payoffs of agents at auction stage at a BNE_{PA} is independent on its auction format, which will help us simplify the computations. In the remaining of this paper, we will assume the existence of BNE_{PA} at the auction stage, and focus on designing optimal strategy for DP stage.

2.2 Data Purchasing Model

At data purchasing stage agents i may acquire a costly signal (data) s_i to refine her knowledge of ω_i (targeting) , with $s_i \in [\underline{s}, \overline{s}]$. Signals received by different buyers are independent. The advertiser can choose the quality of signal, α_i, she buys, with higher α_i indicating a more precise picture of ω_i but also costing more, and $\alpha_i \in [\underline{\alpha}, \overline{\alpha}]$. Agents of the same class $\mu = \eta(i)$ as i have the same cost function $\Phi_\mu(\alpha)$, which is assumed to be public knowledge, satisfying $\Phi_\mu(\underline{\alpha}) = 0$ and is non-decreasing in signal quality α. We interpret Φ_μ as the cost to acquire a certain level of information for i, including like the unit price of data, or i's time or energy cost of data mining on such amount of data. So the cost for the same quality of data may vary across different classes of agents. The qualities of data agents choose to purchase will also be referred as their DP strategies. We will later define and show how to find the equilibrium $(\alpha_1^*, \alpha_2^* \ldots \alpha_N^*)$ in data purchasing stage.

Advertiser i who has data quality α_i will update her belief about ω_i according to Bayes Rule: her knowledge of ω_i updates from $F_{\eta(i)}$ to F_i', with mean v_i updated to v_i' and $\omega_i, v_i, v_i' \in [\underline{\omega}, \overline{\omega}]$. We assume agent i will choose her bidding strategy b_i according to posterior mean v_i', *i.e.*, she submits $b_i(v_i')$ according to some function $b_i(\cdot)$ at auction stage. More precisely, $v_i'(s_i, \alpha_i) \equiv \mathbb{E}[\omega_i | s_i, \alpha_i]$. Notice that the knowledge of v_i' is uncertain before acquiring s_i, so we need to introduce $H_{\alpha_i}(v) = Pr\{v_i'(s_i, \alpha_i) \leq v\}$ as the prior cumulative distribution of v_i' with index v and parameter α_i, and let h_{α_i} be the corresponding density function. For the simplicity of notation, we may interchangeably denote $H_{\alpha_i}(x) = H_i$ in this paper.

2.3 Problem Formulation

Our goal is to properly formulate the problem agents facing at DP stage, to define the notion of the optimal DP strategy, and to show how to calculate such strategy. To handle the first task in this subsection, we now have to trace agents' decision-making process backward from auction stage to DP stage.

Suppose agents choose a DP strategy $\boldsymbol{\alpha} = (\alpha_1, \alpha_2 \ldots \alpha_N)$ given a BNE_{PA} $(b_1^*, b_2^* \ldots b_N^*)$ already have been reached at the SPA. Then from agents i's point of view, by Integral-form Envelope Theorem [38], her expected utility can now be written as [28]

$$\begin{aligned} \mathbb{E}[u_i(b_i^*(v_i'), \mathbf{b}_{-i}^*] &= \sum_{j=1}^{K} c_j \cdot z_{i,j}(v_i') \cdot v_i' - \mathbb{E}[p_i(v_i')] \\ &= \sum_{j=1}^{K} c_j \cdot \int_{\underline{\omega}}^{v_i'} z_{i,j}(t) dt. \end{aligned} \tag{1}$$

Where $z_{i,j}(v_i') = Pr(x_i(\mathbf{b}) = c_j)$ denotes the probability that i obtains j-th slot. This formulation holds true when $\mathbb{E}[p_i(\underline{\omega})] = 0$. Then for condition of agent 1, the probability she wins the first slot is $z_{1,1}(v_1') = \prod_{l=2}^{N} H_l(v_1')$, for the second slot is $z_{1,2} = \sum_{k \neq 1} (1 - H_k(v_1')) \prod_{l \neq 1, k} H_l(v_1') \ldots$. In general,

$$z_{1,j}(v_1') = \sum_{\substack{T, \\ T \subseteq \{2,3,\ldots,N\}, \\ |T| = j-1}} \prod_{k \in T} (1 - H_k(v_1')) \prod_{l \in \{2,3 \cdots N\} \setminus T} H_l(v_1'). \tag{2}$$

And similar derivations for other $z_{i,j}$. To simplify notation we define a auxiliary function $Q_i(t) = \sum_{j=1}^{K} c_j \cdot z_{i,j}(t)$, then equation (1) can be simplified as $\int_{\underline{\omega}}^{v_i'} Q_i(t) dt$.

With the above derivations, we can now consider i's DP strategy. Since v_i' is unknown prior to s_i, we should do expectation of u_i in equation (1) with respect to v_i' :

$$\begin{aligned} \mathbb{E}_{v_i'}[\mathbb{E}[u_i(b_i^*(v_i'), \mathbf{b}_{-i}^*]] &= \mathbb{E}_{v_i'}\left[\int_{\underline{\omega}}^{v_i'} Q_i(t) dt\right] \\ = \int_{\underline{\omega}}^{\overline{\omega}} \left(\int_{\underline{\omega}}^{x} Q_i(t) dt\right) h_{\alpha_i}(x) dx &= \int_{\underline{\omega}}^{\overline{\omega}} (1 - H_{\alpha_i}(x)) Q_i(x) dx. \end{aligned}$$

Considering the expenditure paid during DP and the outcome received during auction, the agents choose their DP strategies according to the following optimization problem:

$$\alpha_i \in \arg\max \int_{\underline{\omega}}^{\overline{\omega}} (1 - H_{\alpha_i}(x)) Q_i(x) dx - \Phi_{\eta(i)}(\alpha_i). \tag{3}$$

Denote the above payoff to be optimized as $\pi_i(\alpha_i, \alpha_{-i})$. Comparing equation (1) and (3), we would find goals of two stages are totally different: for auction stage it is to choose some bidding strategy to maximize utility expectation (1), while for DP stage it is to choose some α to maximize the deterministic payoff (3). So the auction stage should be considered as a Bayesian game while the DP stage is a complete information game. For DP stage, its equilibrium concept is defined as follows.

Definition 2.3 (Nash Equilibrium in Data Purchasing (NE_{DP})). A data purchasing strategy profile $(\alpha_1^*, \alpha_2^* \dots \alpha_N^*)$ forms a Nash Equilibrium if for any i, α', we have $\pi_i(\alpha_i^*, \alpha_{-i}^*) \geq \pi_i(\alpha', \alpha_{-i}^*)$.

Thus, our goal is to derive optimal data purchasing strategy $\boldsymbol{\alpha}^*$ under different scenarios. We start from solving a simple case.

3 GAUSSIAN LEARNING WITH LINEAR COST

In this section, we focus on a simple scenario to demonstrate the basic rationale of finding the optimal DP strategy. In this simple case, there are 2 ad slots and 2 agents, *i.e.*, each agent is guaranteed to win a slot. We describe one representative scheme under the framework developed in Section 2. Specifically, in the auction stage, the payment rule would be VCG mechanism in position auction [21]. Thus, truthful report would be the dominant strategy towards equilibrium state. In consistence with previous economic learning models, in the DP stage Gaussian (GAS) Learning Model is adopted for advertisers, for it nicely quantify the "quality of signals" and is feasible to estimate empirically [15, 23]. We solve this model by proving properties such as the existence and uniqueness of the equilibrium as well as showing how to calculate the optimal DP strategy, for both homogeneous and heterogeneous settings.

3.1 Setup

Since there are only two agents, for the simplicity of notations we will suppress the class indexes and let agents' names represent their own belonging classes: $\eta(1) = 1, \eta(2) = 2$. Agents have Gaussian priors of their valuations: $F_i = \mathcal{N}(v_i, \frac{1}{\beta_i})$, where $\beta_i > 0$ measures the precision of the information i at hand in prior. After purchasing data of quality α_i, agents i receives some private information, works out some data mining, and then obtains $s_i = \omega_i + \epsilon_i, \epsilon_i \sim \mathcal{N}(0, \frac{1}{\alpha_i})$. Here only the summation s_i is observed by i, and the noise term ϵ_i is independent on ω_i. So we can see that the higher quality α_i is acquired, the more precise the signal is. Agents follow a Gaussian Learning and update their beliefs about ω_i according to Bayes Rule: $\omega_i | s_i, \alpha_i \sim \mathcal{N}(v_i', \frac{1}{\alpha_i + \beta_i})$, where $v_i' = \frac{\alpha_i s_i + \beta_i v_i}{\alpha_i + \beta_i}$.

To form the optimization problem, we now have to compute the learning structure H. According to the properties of Gaussian Distribution, it can be calculated that the distribution of v_i' prior to s_i is $\mathcal{N}(v_i, \sigma_i^2)$, where $\sigma_i^2 = \frac{\alpha_i}{\beta_i(\alpha_i + \beta_i)}$. Therefore,

$$H_{\alpha_i}(v) = \int_{-\infty}^{v} \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left\{-\frac{(x - v_i)^2}{2\sigma_i^2}\right\} dx. \tag{4}$$

We assume the cost is linear: $\Phi_i(\alpha_i) = \phi_i(\alpha_i - \underline{\alpha}), \phi_i > 0$.

From now on we start to consider the problem from agents 1's point of view. Corresponding to equation (1), the expected utility for 1 at auction stage when truthful report is

$$\mathbb{E}[u_1(v_1')] = c_1 \int_{-\infty}^{v_1'} H_2(x)dx + c_2 \int_{-\infty}^{v_1'} 1 - H_2(x)dx. \tag{5}$$

The following lemma shows that there always exists an equilibrium as long as their prior means are the same, which means equilibrium is reachable for agents with similar beliefs.

Lemma 3.1. *If $v_1 = v_2$, then there exists a NE_{DP}.*

Proof. By equation (4) we obtain

$$\frac{\partial H_1(v)}{\partial \alpha_1} = -\frac{v - v_1}{2\sqrt{2\pi}} \exp\left\{-\frac{(v - v_1)^2}{2\sigma_1^2}\right\} \sqrt{\frac{\beta_1^3}{\alpha_1^3(\alpha_1 + \beta_1)}},$$

And notice that

$$\begin{aligned}\frac{\partial \pi_1}{\partial \alpha_1} &= -\int_{-\infty}^{+\infty} \frac{\partial H_1(v)}{\partial \alpha_1} Q_1(v)dv - \phi_1 \\ &= \frac{(c_1 - c_2) \cdot (\sigma_1^2 + \sigma_2^2)^{-\frac{1}{2}}}{2\sqrt{2\pi}(\alpha_1 + \beta_1)^2} \exp\left\{-\frac{(v_1 - v_2)^2}{2(\sigma_1^2 + \sigma_2^2)}\right\} - \phi_1,\end{aligned} \tag{6}$$

combining with equation (3) (5), it can be derived that

$$\begin{aligned}\frac{\partial^2 \pi_1}{\partial \alpha_1^2} = &-\frac{c_1 - c_2}{2\sqrt{2\pi}(\alpha_1 + \beta_1)^4} \frac{1}{\sqrt{\sigma_1^2 + \sigma_2^2}} \exp\left\{-\frac{(v_1 - v_2)^2}{2(\sigma_1^2 + \sigma_2^2)}\right\} \\ &\left[2(\alpha_1 + \beta_1) + \frac{\sigma_1}{\sigma_1^2 + \sigma_2^2}\left(1 - \frac{(v_1 - v_2)^2}{\sigma_1^2 + \sigma_2^2}\right)\right].\end{aligned}$$

And the same form for agents 2. So it can be observed that when $v_1 = v_2$, we have π_i being strictly concave in α_i. By Proposition 8.D.3 in [37], since the strategy space for every agents is $[\underline{\alpha}, \overline{\alpha}]$, which is a nonempty, convex and compact subset of Euclidean space, combining with that π_i is continuous in (α_1, α_2) and concave in α_i, there exists a Nash equilibrium. □

3.2 Homogeneous Agents

In this subsection, we restrict agents to be homogeneous, meaning both of them belong to the same class. *i.e.*, $v_1 = v_2 = v, \beta_1 = \beta_2 = \beta, \phi_1 = \phi_2 = \phi$. We claim there is one and only one equilibrium in this setting, and we also show how to derive such purchasing strategy in the proof.

Theorem 3.2. *For 2 homogeneous agents, 2 slots with GAS Learning and linear cost, there exists a symmetric and unique NE_{DP}.*

Proof. By lemma 3.1, there must exist a NE_{DP} when $v_1 = v_2 = v$. Denote $\nu_i(\alpha_1, \alpha_2) = \frac{(c_1 - c_2)\cdot(\sigma_1^2 + \sigma_2^2)^{-\frac{1}{2}}}{2\sqrt{2\pi}(\alpha_i + \beta_i)^2}$, then we check the *Karush-Kuhn-Tucker* (KKT) first order condition for agents i's problem,

$$\begin{cases} \frac{\partial \pi_i(\alpha_1, \alpha_2)}{\partial \alpha_i} & = & \nu_i(\alpha_1, \alpha_2) - \phi = -\lambda_i + \gamma_i \\ \lambda_i(\alpha_i - \underline{\alpha}) & = & 0 \\ \gamma_i(\alpha_i - \overline{\alpha}) & = & 0 \\ \lambda_i, \gamma_i & \geq & 0 \end{cases}, \tag{7}$$

here λ_i and γ_i are the Lagrange multipliers for restrictions $\alpha_i \geq \underline{\alpha}$ and $\alpha_i \leq \overline{\alpha}$ respectively.

Suppose there exists an asymmetric equilibrium (α_1^*, α_2^*), *w.l.o.g.* assuming that $\alpha_1^* < \alpha_2^*$. This implies $\alpha_1^* < \overline{\alpha}$ and $\alpha_2^* > \underline{\alpha}$. Then by (7), $\gamma_1 = 0$ and $\lambda_2 = 0$. So we have $\frac{\partial \pi_1(\alpha_1^*, \alpha_2^*)}{\partial \alpha_1} = -\lambda_1 \leq 0$ and $\frac{\partial \pi_2(\alpha_1^*, \alpha_2^*)}{\partial \alpha_2} = \gamma_2 \geq 0$. Then $\phi = \Phi_1'(\alpha_1^*) \geq \nu_1(\alpha_1^*, \alpha_2^*) > \nu_2(\alpha_1^*, \alpha_2^*) \geq \Phi_2'(\alpha_2^*) = \phi$. Which is a contradiction. So $\alpha_1^* = \alpha_2^*$.

Now we prove the uniqueness of the equilibrium. First we show the interior equilibrium is unique. By (7), $\frac{\partial \pi_i}{\partial \alpha_i} = 0$ for the interior equilibrium. Since we have proved the equilibrium must be symmetric, this shows that $\frac{\partial \pi_i(\alpha, \alpha)}{\partial \alpha} = 0$. *i.e.*, $\nu_i(\alpha, \alpha) - \phi = 0$. Since σ_i is increasing in α_i and $\nu_i(\alpha, \alpha)$ is decreasing in α, therefore, it guarantees the uniqueness of interior equilibrium.

Then let us look at the corner equilibrium. There is only two possible corner equilibriums $(\overline{\alpha}, \overline{\alpha})$ and $(\underline{\alpha}, \underline{\alpha})$. Suppose these two equilibriums exist simultaneously, by (7) we have $\frac{\partial \pi_i}{\partial \alpha_i} = -\lambda_i \leq 0$ at $(\underline{\alpha}, \underline{\alpha})$ and $\frac{\partial \pi_i}{\partial \alpha_i} = \gamma_i \geq 0$ at $(\overline{\alpha}, \overline{\alpha})$. But this implies $\phi \geq \nu_i(\overline{\alpha}, \overline{\alpha}) > \nu_i(\underline{\alpha}, \underline{\alpha}) \geq \phi$. Since $\underline{\alpha} < \overline{\alpha}$, this yields a contradiction. So the corner equilibrium must be unique.

Finally we show the interior equilibrium and corner equilibrium cannot exist concurrently. *W.l.o.g.*, suppose there is an interior equilibrium (α^*, α^*) and a corner equilibrium $(\underline{\alpha}, \underline{\alpha})$. Then we have $\frac{\partial \pi_i}{\partial \alpha_i} = 0$ at (α^*, α^*) and $\frac{\partial \pi_i}{\partial \alpha_i} = \gamma_i \geq 0$ at $(\underline{\alpha}, \underline{\alpha})$. But we again see that $\phi = v_i(\alpha^*, \alpha^*) < v_i(\underline{\alpha}, \underline{\alpha}) = \phi$, contradicting to $\underline{\alpha} < \alpha^*$. So the corner equilibrium and the interior equilibrium cannot both exist.

Therefore, we have completed the proof that the equilibrium must be symmetric and unique. □

Under homogeneous setting, we can observe that only prior precision β and marginal cost ϕ affect the interior equilibrium α^*. Since the analytic form of v is provided, we can calculate the optimal DP strategy in equation $v(\alpha) - \phi = 0$, simply by resorting to classical root-finding algorithms, such as Newton's method or Secant method. The relation between α^* with β, ϕ are drawn in Figure 2. We can observe that for fixed ϕ, α first increases with β then decreases, showing the trade-offs agents have to make between enhancing the precision of knowledge and paying for such acquisitions. Also agents will tend to purchase less data for higher marginal cost, confirming intuition.

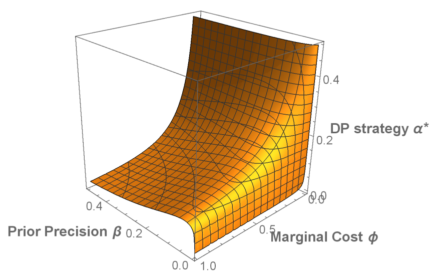

Figure 2: Homogeneous agents. $c_1 = 1, c_2 = 1/2$

Furthermore, if we view ϕ as the unit price of cookies and let $Rev = 2\phi\alpha^*$ be the revenue of the platform provider, we can determine the corresponding revenue-maximization price for Data Market by a simple first-order derivation.

Proposition 3.3 (Revenue-Maximization Price). *The revenue-maximization price for a Data Market with 2 homogeneous agents is* $\phi^* = \frac{(c_1 - c_2)}{6\sqrt{3\pi\beta}}$, *with corresponding optimal revenue* $Rev^* = \frac{(c_1-c_2)\sqrt{\beta}}{6\sqrt{3}\pi}$ *and equilibrium state* $\alpha^* = \frac{\beta}{2}$.

3.3 Heterogeneous Agents

In this subsection, we consider two directions of modeling heterogeneous agents. More concretely, we restrict that $v_1 = v_2 = v$, and their classes differ only in that either $\phi_1 \neq \phi_2$, or $\beta_1 \neq \beta_2$. For these heterogeneous settings, it is intuitive that (1) agent with higher precision of prior knowledge will acquire less data when their cost functions are the same, or (2) agent who has a higher marginal cost of acquiring data (for example, poor data mining technology) will buy less even their prior beliefs are the same. We will first formally define these intuitions and verify them through a detailed and rigorous analysis.

Definition 3.4 (Intuitive Equilibrium). A profile of (α_1^*, α_2^*) forms an intuitive equilibrium if $\alpha_1^* \geq \alpha_2^*$ under condition when $\phi_1 < \phi_2$ and $\beta_1 = \beta_2$, or condition when $\phi_1 = \phi_2$ and $\beta_1 < \beta_2$, vice versa.

Theorem 3.5. *For 2 heterogeneous agents, 2 slots with GAS Learning and linear cost, there exists a unique NE_{DP}, and it must be intuitive.*

Proof. First we prove that any equilibrium (α_1^*, α_2^*) must be intuitive. Denote $v_i(\alpha_1, \alpha_2, \beta_1, \beta_2) = \frac{(c_1-c_2)\cdot(\sigma_1^2+\sigma_2^2)^{-\frac{1}{2}}}{2\sqrt{2\pi}(\alpha_i+\beta_i)^2}$.

Consider when $\beta_1 = \beta_2 = \beta$ but $\phi_1 < \phi_2$. Suppose $\alpha_1^* < \alpha_2^*$. Then it implies $\alpha_1^* < \overline{\alpha}$ and $\alpha_2^* > \underline{\alpha}$. Then $\frac{\partial \pi_1}{\partial \alpha_1} = -\lambda_1 \leq 0$ and $\frac{\partial \pi_2}{\partial \alpha_2} = \gamma_2 \geq 0$, so again we have $\phi_2 \leq v_2(\alpha_1^*, \alpha_2^*, \beta, \beta) < v_1(\alpha_1^*, \alpha_2^*, \beta, \beta) \leq \phi_1$, a contradiction, so we must have $\alpha_1^* \geq \alpha_2^*$ when $\phi_1 < \phi_2$.

Then consider another case when $\phi_1 = \phi_2 = \phi$ but $\beta_1 < \beta_2$. Similarly we can derive that $\phi \leq v_2(\alpha_1^*, \alpha_2^*, \beta_1, \beta_2) < v_1(\alpha_1^*, \alpha_2^*, \beta_1, \beta_2) \leq \phi$. It is also a contradiction. So $\alpha_1^* \leq \alpha_2^*$ when $\beta_1 > \beta_2$.

Finally we prove the uniqueness of equilibrium. The uniqueness of corner equilibrium can be proved similar to Theorem 3.2. Consider an interior equilibrium (α_1^*, α_2^*) which must satisfy

$$\frac{(c_1 - c_2) \cdot (\sigma_1^2 + \sigma_2^2)^{-\frac{1}{2}}}{2\sqrt{2\pi}(\alpha_i + \beta_i)^2} - \phi_i = 0, i = 1, 2.$$

Comparing the forms of $i = 1, 2$ we have

$$\sqrt{\phi_1}(\alpha_1 + \beta_1) = \sqrt{\phi_2}(\alpha_2 + \beta_2), \tag{8}$$

It serves as a constraint for an equilibrium (α_1^*, α_2^*). Suppose there exists another interior equilibrium (α_1', α_2'), where $\alpha_1' < \alpha_1^*$. By equation (8) we have $\alpha_2' < \alpha_2^*$. Then $v_i(\alpha_1', \alpha_2', \beta_1, \beta_2) > v_i(\alpha_1^*, \alpha_2^*, \beta_1, \beta_2) = \phi_i$. Which implies (α_1', α_2') is not an interior equilibrium. Then there is only one interior equilibrium. The interior equilibrium and corner equilibrium cannot exist concurrently for the same reason described in Theorem 3.2. So we now have completed the proof. □

The optimal DP strategy can also be calculated by applying root-finding algorithms to equations $v_i - \phi_i = 0$. From Figure 3, we can observe that one's optimal DP strategy would increase with her adversary's prior precision and marginal cost.

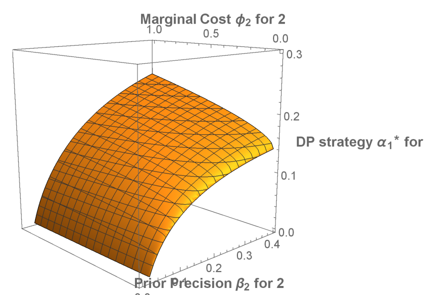

Figure 3: Heterogeneous agents. $c_1 = 1, c_2 = 1/2, \beta_1 = 0.2, \phi_1 = 0.4$.

4 GENERAL LEARNING MODEL WITH CONVEX COST

In this section, we consider a more general scheme for any $N \geq 2, K \geq 1$. Here we assume the prior valuation distributions to be homogeneous for all agents: $F_{\eta(i)} = F$. We extend the linear cost model adopted in Section 3 to the space of all convex functions $\Phi_{\eta(i)}$, which captures the fact that valuable information becomes rare and harder to find as more efforts are exerted or time is wasted.

The difference between distinct classes of agents would be reflected by their cost functions $\Phi_{\eta(i)}$. The learning structure H_{α_i} follows the same form for all agents.

First we focus on a homogeneous setting where $\Phi_{\eta(i)} = \Phi$. The following theorem shows that, as long as the learning process H is strict log-convex with respect to α, the existence, the symmetry as well as the uniqueness of equilibrium are assured. The intuition of building such learning structure is that with more data, the relative probability of information gain from DP would be larger.

THEOREM 4.1. *For an SPA $\mathscr{A}$, if H_{α_i} is strict log-convex with respect to α_i, i.e., $\frac{\partial^2 \log H_{\alpha_i}}{\partial \alpha_i^2} > 0$, then there exists a symmetric and unique NE_{DP} for homogeneous agents.*

We refer to Appendix A for the detailed proof of this theorem. From the proof, we can find the purchasing strategy can be obtained by calculating an equation $W_i(\alpha) = 0$ via root-finding algorithms.

We next consider a heterogeneous setting, where marginal cost function $\Phi'_{\eta(i)}$ may vary across different agents. We prove that classes of higher marginal cost will always acquire less information at an NE_{DP}, as one direction of generalization for Theorem 3.5.

Definition 4.2 (General Intuitive Equilibrium). An NE_{DP} is generally intuitive if it satisfies that agents of the same class acquire the same quality of data: $\forall i\,(\mu = \eta(i)) \Rightarrow \exists \alpha_\mu (\alpha_i^* = \alpha_\mu)$. Moreover, class with larger marginal cost will acquire less data: $\forall i, g\,(\mu = \eta(i)) \wedge (\delta = \eta(g)) \wedge \forall \alpha \left(\Phi'_\mu(\alpha) > \Phi'_\delta(\alpha)\right) \Rightarrow \alpha_i^* \leq \alpha_g^*$.

THEOREM 4.3. *For an SPA $\mathscr{A}$, if H_{α_i} is log-convex with respect to α_i, i.e., $\frac{\partial^2 \log H_{\alpha_i}}{\partial \alpha_i^2} > 0$, then if NE_{DP} exists, it must be intuitive for heterogeneous agents.*

The proof of Theorem 4.3 is provided in Appendix B.

A representative learning model is the Truth-or-Noise learning, which we are going to formally define below. It is easy to verify its log-convexity. As indicated by the name, in this model the quality of signal is interpreted as the probability that an agent obtains the ground-truth of ω, which is also consistence with existing works concerning targeting [9].

Definition 4.4 (Truth-or-Noise Learning Model). In truth-or-noise learning model, priors are uniform distributions on $[\underline{\omega}, \overline{\omega}]$. In other words, $F(x) = \frac{x-\underline{\omega}}{\overline{\omega}-\underline{\omega}}, x \in [\underline{\omega}, \overline{\omega}]$. $F(x) = 0$ when $x < \underline{\omega}$ and $F(x) = 1$ when $x > \overline{\omega}$. The quality of signal $\alpha \in (1/2, 1]$. Having purchasing α, one may obtain just her precise ω_i with probability α, or a noise signal of sample mean $\underline{\omega}$ with probability $1 - \alpha$.

5 NUMERICAL RESULTS

In this section, we report our numerical results on how agents react to different strategic environments. We consider three types of CTRs: $c_i = 2^{-(i-1)}$, $c_i = i^{-1}$, and $c_i = (\log_2(i+1))^{-1}$. We name them as EXP-CTR, HAM-CTR, and LOG-CTR, in decreasing order of discounting effects. CTRs that are less discounted may stand for a more popular online website. The agents in the evaluation are configured to be homogeneous.

We will mainly investigate on GAS Learning and ToN Learning agents. We implement classical Newton's algorithm to find the optimal strategies. Having examined different combinations of parameters, we found that normally GAS Learning agents may display certain properties within relatively small N while ToN is more suitable for simulating environment where more agents are participating. We will next demonstrate our findings under representative combinations of parameters in the following evaluations.

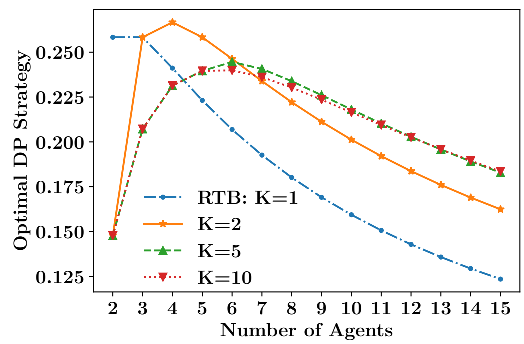

Figure 4: GAS Learning. Comparison on number of slots. Fixed EXP-CTR.

For GAS Learning, we vary the number of agents from 2 to 15 with step 1. We fix at a point $\beta = 0.2, \phi = 0.4$. We first fix at EXP-CTR in Figure 4. It shows that except for RTB case $K = 1$ that the α^* is decreasing with N, generally for other cases the α^* first increases with N up to a maximal point and then decreases. This is due to the trade-off between the revenue brought by improving the valuations precision and the loss ensued by fiercer competition. Another tendency is that with more competitors coming in and less ad slots become available, the agents would tend to purchase less data. This is due to that agents are trying to avoid the risks of losing the auction as the environment becomes more competitive even after purchasing huge amount of data, which may bring only large wasted data expenditure to the agents.

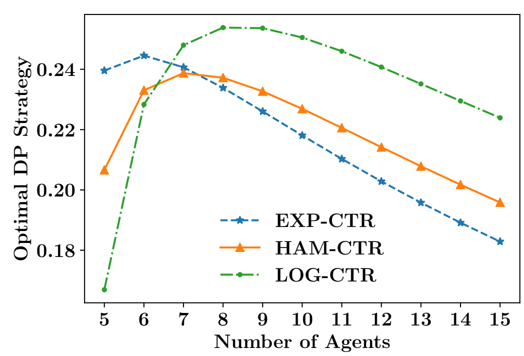

Figure 5: GAS Learning. Comparison on CTRs. $K = 5$

In Figure 5 we fix $K = 5$. The curvatures show that for GAS Learning agents, the uphills of optimal strategies ascend steeper and the downhills descend more gently, in websites with less discounting effects. And in the long run as more competitors participate the auction, agents tend to purchase more data in LOG-CTR than in EXP-CTR. This illustrates the incentive effect that CTRs bring to the agents. It can be interpreted as that in a popular online environment, such as one with LOG-CTR, agents are more likely to receive more ad clicks that induces larger profits, than one with EXP-CTR. Thus agents behave as they want to take chances to obtain higher revenue in a popular website by purchasing enough amount of data.

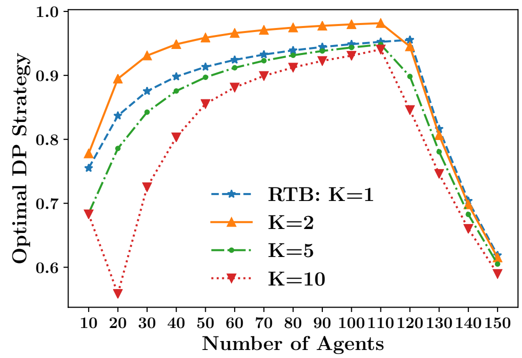

Figure 6: ToN Learning. Comparison on number of slots. Fixed EXP-CTR.

For ToN Learning, we set the number of agents from 10 to 150 with step 10. We let $\phi = 2$. First look at Figure 6. Pay attention to that unlike GAS Learning, for ToN learning, when K is a even number it will encounter a sudden drop at point $N = K + 2$. And for the same N the optimal α^* may oscillate between adjacent even and odd K. But generally, the overall tendency of optimal DP strategies with respect to N still tends to decline when K becomes much larger, which again confirms the trade-off agents have to make between profits made in auction and risk of wasted data purchasing.

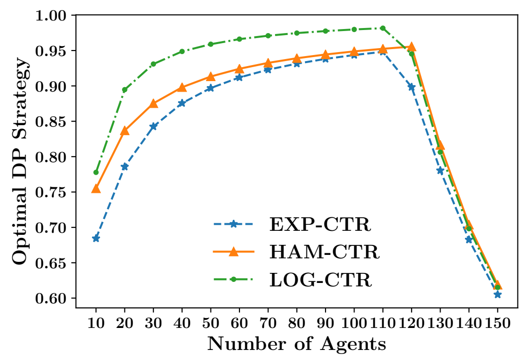

Figure 7: ToN Learning. Comparison on CTRs. $K = 5$

In Figure 7 we can observe that for ToN learning agents they would purchase most data in LOG-CTR while least in EXP-CTR, further illustrating the incentive effect in this scenario. But from the graphs we can also notice that the turning point tends to occur at a larger N after where the optimal DP strategy decreases more rapidly as more agents are involved. So ToN learning agents normally may be suitable for modeling advertisers facing larger number of competitors.

6 RELATED WORKS

In this section, we briefly review literatures about data usage in an auction context.

Our work is closely related to previous ones which also considered the role of data in auctions. Specifically, [30] studied how would improved targeting affect the revenue when facing different number of advertisers. [9] researched the value of data for different advertisers with different valuations or budgets. [24] designed an optimal mechanism when considering data usage, which might bring additional revenue. [22] provides an optimal signaling scheme for revenue maximization for a second price auction. [4] designed an optimal mechanism for selling data by assuming an one-round protocol. [48] designed strategy-proof data auctions considering negative externalities. Recent works [5, 20, 40] highlight theoretical progress about targeting and signaling in ad auctions.

How to choose proper strategy for obtaining costly signals is the main focus on topic of information acquisition [8, 18], whose framework naturally fits data usages in auctions. [44] designed an optimal mechanism considering acquisition process, which casted insight on our framework design. [34] considered the optimal acquisition strategy for one item in vickery auction, while ours extends to a wide classes of ad auction with any number of ad slots. Recent work [27] propose the optimal and efficient mechanisms with dynamic acquisition.

The authors in [7] considered the interdependent relation between the valuations for acquisition, addressing the role of information externalities in an auction. This issue is particular important in ad auction since advertisers' valuations toward ad slots may be interdependent to each other and satisfy common-value model. The roles of information asymmetries in common-value vickery auction was addressed in [1], where the complex condition of equilibriums was refined to as a new concept called TRE. [45] considered two asymmetrically informed bidders in a common-value auction with discrete signals and give the characterization of equilibrium. The authors in [10] derived when would the agents choose to observe the signal under certain interdependent structure. [25] proposes an approximation algorithm for winner determination under externalities. [36] researched the effect of information externalities in GSP mechanism, which were naturally raised when considering data usage. Nevertheless, since in our model the signaling process is modeled as a complete information game while advertisers' valuation are assumed to be independent, we do not concern consider these issues and defer them for future works.

7 CONCLUSION

In this paper, we have considered the data purchasing problem faced by advertisers before an ad auction. Having properly formulated the problem and the objective, we started with a simple scenario and have solved it through rigorous mathematical analysis. The intuitions have been extended to a more general scheme which embraces a wide class of learning agents. Our numerical results have revealed the relations between the optimal strategies with different configurations of the strategic environment.

A PROOF OF THEOREM 4.1

Proof. First we prove the existence of equilibrium. The log-convex constraint of H_{α_i} is equivalent to:

$$\frac{\partial^2 \log H_{\alpha_i}}{\partial \alpha_i^2} = \frac{1}{H_i^2}\left[\frac{\partial^2 H_i}{\partial \alpha_i^2} H_i - (\frac{\partial H_i}{\partial \alpha_i})^2\right] > 0. \tag{9}$$

Then we have $\frac{\partial^2 H_i}{\partial \alpha_i^2} > 0$ for all α_i. And so H_{α_i} is strictly convex in α_i. Thus,

$$\frac{\partial^2 \pi_i(\alpha_i)}{\partial \alpha_i^2} = -\int_{\underline{\omega}}^{\overline{\omega}} \frac{\partial^2 H_i(v)}{\partial \alpha_i^2} Q_i(v) dv < 0. \tag{10}$$

Next we look at the KKT condition for bidder i, denote $v_i(\alpha_i, \alpha_{-i}) = -\int_{\underline{\omega}}^{\overline{\omega}} \frac{\partial H_{\alpha_i}(v)}{\partial \alpha_i} Q_i(v) dv$, we have

$$\begin{cases} \frac{\partial \pi_i(\alpha_i, \alpha_{-i})}{\partial \alpha_i} & = & v_i(\boldsymbol{\alpha}) - \Phi'(\alpha_i) & = -\lambda_i + \gamma_i \\ \lambda_i(\alpha_i - \underline{\alpha}) & = & 0 \\ \gamma_i(\alpha_i - \overline{\alpha}) & = & 0 \\ \lambda_i, \gamma_i & \geq & 0 \end{cases} \tag{11}$$

We first prove the equilibrium must be symmetric. Suppose $\alpha_1^* < \alpha_2^*$, which implies $\alpha_1^* < \overline{\alpha}$ and $\alpha_2^* > \underline{\alpha}$. We have $\gamma_1 = 0$ and $\lambda_2 = 0$. Thus, $\frac{\partial \pi_1(\alpha_1^*, \alpha_2^*, \alpha_{-1,2})}{\partial \alpha_1} = -\lambda_1 \leq 0$ and $\frac{\partial \pi_2(\alpha_1^*, \alpha_2^*, \alpha_{-1,2})}{\partial \alpha_2} = \gamma_2 \geq 0$. Then we obtain $\Phi'(\alpha_1^*) \geq v_1(\alpha_1^*, \alpha_2^*, \alpha_{-1,2}^*)$ and $\Phi'(\alpha_2^*) \leq v_2(\alpha_1^*, \alpha_2^*, \alpha_{-1,2}^*)$. But we will prove $v_1(\alpha_1^*, \alpha_2^*, \alpha_{-1,2}^*) > v_2(\alpha_1^*, \alpha_2^*, \alpha_{-1,2}^*)$ which leads to $\Phi'(\alpha_2^*) < \Phi'(\alpha_1^*)$.

Look at quantity

$$\begin{aligned} & v_1(\alpha_1^*, \alpha_2^*, \alpha_{-1,2}^*) - v_2(\alpha_1^*, \alpha_2^*, \alpha_{-1,2}^*) \\ = & -\int_{\underline{\omega}}^{\overline{\omega}} \left[\frac{\partial H_{\alpha_1^*}(v)}{\partial \alpha_1^*} Q_1(v) - \frac{\partial H_{\alpha_2^*}(v)}{\partial \alpha_2^*} Q_2(v) \right] dv. \end{aligned} \tag{12}$$

We want to transform the integrated part in (12) in a more explicit form. Recall the definition of Q_i in Section 2.3 we can observe that it is a summation whose terms include a series of product of form H and $1-H$. What we do is simply rearrange the production to "match" terms of 1, 2. For example, term

$$\frac{\partial H_1(v)}{\partial \alpha_1^*} \prod_{n \neq 1} H_n - \frac{\partial H_2(v)}{\partial \alpha_2^*} \prod_{n \neq 2} H_n = \prod_{n=1}^{N} H_n \left[\frac{\frac{\partial H_1}{\partial \alpha_1^*}}{H_1} - \frac{\frac{\partial H_2}{\partial \alpha_2^*}}{H_2} \right]$$

The similar procedures can be applied to every matched terms. So we will eventually turn equation (12) into a summation of terms of form

$$-M \cdot \int_{\underline{\omega}}^{\overline{\omega}} \left[\frac{\partial H_1}{\partial \alpha_1^*} \frac{1}{g_i(H_1)} - \frac{\partial H_2}{\partial \alpha_2^*} \frac{1}{g_i(H_2)} \right] dv, i = 1, 2. \tag{13}$$

Here $g_1(H) = H$ and $g_2(H) = 1-H$, and $M > 0$ is a series production of form of $1-H$ and H independent of 1, 2. The strict log-convex of H is equivalent to that $\frac{\partial H_{\alpha_i}(v)}{\partial \alpha_i} \frac{1}{f_1(H_i)}$ is strictly increasing. And by (9) we can conclude form of $\frac{\partial H_{\alpha_i}(v)}{\partial \alpha_i} \frac{1}{f_2(H_i)}$ are also increasing just by verifying the positiveness of its first derivative on α_i:

$$\frac{\partial}{\partial \alpha_i} \left[\frac{\partial H_{\alpha_i}(v)}{\partial \alpha_i} \frac{1}{1 - H_{\alpha_i}(v)} \right] = \frac{1}{(1-H_i)^2} \left[\frac{\partial^2 H_i}{\partial \alpha_i^2}(1 - H_i) + (\frac{\partial H_i}{\partial \alpha_i})^2 \right] > 0. \tag{14}$$

Therefore, terms of form (13) are all positive. Thus, $v_1(\alpha_1^*, \alpha_2^*, \alpha_{-1,2}^*) > v_2(\alpha_1^*, \alpha_2^*, \alpha_{-1,2}^*)$. However, this indicates $\Phi'(\alpha_2^*) < \Phi'(\alpha_1^*)$ which implies $\alpha_1^* > \alpha_2^*$, a contradiction. So the equilibrium must be symmetric.

We now prove the equilibrium must be unique. First we show the interior equilibrium must be unique. For interior equilibrium $\boldsymbol{\alpha} = (\alpha^*, \alpha^* \ldots \alpha^*)$, $v_i(\boldsymbol{\alpha^*}) - \Phi'(\alpha^*) = 0$. Denote $W_i(\alpha^*) = v_i(\boldsymbol{\alpha^*}) - \Phi'(\alpha^*)$. What we'll prove is that $W_i(\alpha)$ is monotonically decreasing.

Notice $W_i'(\alpha^*) = \sum_{n=1}^{N} \frac{\partial v_i(\boldsymbol{\alpha^*})}{\partial \alpha_n} - \Phi''(\alpha^*)$. Recall we assume that $\Phi'' \geq 0$. And by (10) $\frac{\partial v_i(\boldsymbol{\alpha^*})}{\partial \alpha_i} < 0$. For $i \neq n$, *w.l.o.g.*, let $i = 1, n = 2$, notice that

$$\begin{aligned} \frac{\partial v_1(\boldsymbol{\alpha})}{\partial \alpha_2} &= -\int_{\underline{\omega}}^{\overline{\omega}} \frac{\partial H_1(v)}{\partial \alpha_1} \frac{\partial Q_1(v)}{\partial \alpha_2} dv \\ &= -\int_{\underline{\omega}}^{\overline{\omega}} \frac{\partial H_1(v)}{\partial \alpha_1} \frac{\partial H_2(v)}{\partial \alpha_2} \left(\sum_{k=1}^{K} c_k(-R_{k-1}(v) + R_k(v)) \right) dv \\ &= -\int_{\underline{\omega}}^{\overline{\omega}} \left(\frac{\partial H_1(v)}{\partial \alpha_1} \right)^2 \left(\sum_{k=2}^{K+1} (c_{k-1} - c_k) R_{k-1}(v) \right) dv < 0 \end{aligned}$$

Where $R_i = \sum_{\substack{T, \\ T \subseteq \{3,4 \cdots N\}, \\ |T| = i-1}} \prod_{\in T} (1 - H_n) \prod_{l \in \{3,4 \cdots N\} \setminus T} H_l$, for $1 \leq i \leq K$ and $R_0 = 0$.

The idea of the above transformation can be concretely seen from the following example:

$$\begin{aligned} & \frac{\partial}{\partial \alpha_2} \left\{ c_1 \prod_{l=2}^{N} H_l + c_2 \sum_{n=2}^{N} (1 - H_n) \prod_{l \neq n, 1} H_l + \ldots \right\} \\ = & \frac{\partial H_2}{\partial \alpha_2} \cdot \left[c_1 \prod_{l=3}^{K} H_l + c_2 \left(-\prod_{l=3}^{N} H_l + \sum_{n=3}^{N} (1 - H_n) \prod_{l \neq n, 1, 2} H_l \right) + \ldots \right] \\ = & \frac{\partial H_2}{\partial \alpha_2} \cdot \left[(c_1 - c_2) \prod_{l=3}^{N} H_l + (c_2 - c_3) \sum_{n=3}^{N} (1 - H_n) \prod_{l \neq n, 1, 2} H_l + \ldots \right] \end{aligned}$$

So we can see that the order of CTRs plays an important rule in our proof. Thus, $\frac{\partial v_i(\boldsymbol{\alpha^*})}{\partial \alpha_n} < 0$ for $i \neq n$. And so $W_i'(\alpha^*) < 0$. The monotonicity shows the uniqueness of symmetric equilibrium.

We now move to the proof of the uniqueness of corner equilibrium. There are only two possible corner equilibriums: $(\underline{\alpha}, \underline{\alpha} \ldots \underline{\alpha})$ and $(\overline{\alpha}, \overline{\alpha} \ldots \overline{\alpha})$. By KKT condition (11), we have $v_i(\underline{\alpha}, \underline{\alpha} \ldots \underline{\alpha}) \leq \Phi'(\underline{\alpha})$ for equilibrium $(\underline{\alpha}, \underline{\alpha} \ldots \underline{\alpha})$ and $v_i(\overline{\alpha}, \overline{\alpha} \ldots \overline{\alpha}) \geq \Phi'(\overline{\alpha})$ for equilibrium $(\overline{\alpha}, \overline{\alpha} \ldots \overline{\alpha})$. By (9) we proved $v_i(\alpha_i, \alpha_{-i})$ is strictly decreasing in α_i, we have $v_i(\overline{\alpha}, \overline{\alpha} \ldots \overline{\alpha}) > v_i(\underline{\alpha}, \underline{\alpha} \ldots \underline{\alpha})$, so two possible equilibriums cannot exist concurrently.

Last we show there is only one possible equilibrium, either interior or corner. *W.l.o.g.* suppose $(\underline{\alpha}, \underline{\alpha} \ldots \underline{\alpha})$ and $(\alpha, \alpha \ldots \alpha)$ both exist. Then we have $v(\alpha, \alpha \ldots \alpha) = \Phi'(\alpha)$ and $v(\underline{\alpha}, \underline{\alpha} \ldots \underline{\alpha}) \leq \Phi'(\underline{\alpha})$. But this implies $\Phi'(\alpha) = v(\alpha, \alpha \ldots \alpha) < v(\underline{\alpha}, \underline{\alpha} \ldots \underline{\alpha}) \leq \Phi'(\underline{\alpha})$ which means $\alpha < \underline{\alpha}$, a contradiction.

We now have completed all the proof that the NE_{DP} must be symmetric and unique. □

B PROOF OF THEOREM 4.2

Proof. by Theorem 4.1, bidders of the same type must acquire the same quality of information.

Next we consider bidders in different groups of type. Consider class μ,δ where $\Phi_\mu'(\alpha) > \Phi_\delta'(\alpha)$, and bidder i in type μ and g in type δ. If they acquire the same quality of information α^*, by equation (11) we have $\Phi_\mu'(\alpha^*) = v_i(\boldsymbol{\alpha^*}) = v_g(\boldsymbol{\alpha^*}) = \Phi_\delta'(\alpha^*)$ since their beliefs are identical in prior. But we have assumed $\Phi_\mu'(\alpha) > \Phi_\delta'(\alpha)$, so it is a contradiction. Therefore, the quality of information acquired by different type of bidders must be different.

Suppose in this condition $\alpha_\mu^* > \alpha_\delta^*$. Then by Theorem 4.1 we obtain that $\Phi_\mu'(\alpha_\mu^*) = v_i(\alpha^*) < v_g(\boldsymbol{\alpha^*}) = \Phi_\delta'(\alpha_\delta^*)$ which contradicts to our assumption. So the equilibrium must be intuitive. □

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